2020

MATHEMATICS

[GENERAL]

Paper : III

[SUPPLEMENTARY]

Full Marks : 100 Time : 3 Hours

The figures in the right-hand margin indicate marks.

Candidates are required to give their answers in their own words as far as practicable.

Symbols have their usual meanings.

GROUP–A

(Linear Programming and Game Theory)

[Marks : 50]

1. Answer any **four** questions: 1×4=4

   a) What is a balanced transportation problem?

   b) Define two person zero-sum game.

   c) Is the union of two convex sets a convex set? Justify your answer.

   d) Give an example of a convex set that has no extreme point.

   e) Express (7, 11) as a linear combination of $\alpha = (2, 3)$ and $\beta = (3, 5)$.

[Turn over]

   f) Define a non-degenerate basic feasible solution.

2. Answer any **six** questions: 2×6=12

   a) Find the extreme points, if any, of the set

   $$S = \{(x, y) \mid x^2 + y^2 \le 25\}.$$

   b) Prove that a hyperplane is a convex set.

   c) Show that (3, 0, 2), (7, 0, 9) and (4, 1, 2) form a basis in E^3.

   d) Show that the vectors (1, 2, 3) and (4, –2, 7) are linearly independent.

   e) Prove that $\max_i \min_j a_{ij} \le \min_j \max_i a_{ij}$.

   f) Construct the dual of the following L.P.P.

   Maximize $Z = 3x_1 + 4x_2$

   subject to $x_1 + x_2 \le 12$,

   $2x_1 + 3x_2 \le 21$,

   $x_1 \le 8,\ x_2 \le 6,\ x_1, x_2 \ge 0$.

   g) In a game with the 2×2 pay-off matrix

a	b
c	d

   where $a < d < b < c$, show that there is no saddle point.

h) If an LPP has two feasible solutions, prove that it has an infinite number of solutions.

i) Is an assignment problem a transportation problem? Justify.

3. Answer any **four** questions: 6×4=24

a) Apply simplex method to solve the L.P.P.:

Maximize $Z = 2x_1 - 3x_2$

subject to $2x_1 + 5x_2 \geq 10$,

$3x_1 + 8x_2 \leq 24$,

$x_1, x_2 \geq 0$.

b) Use duality to solve the L.P.P.:

Minimize $Z = 3x_1 + x_2$

subject to $2x_1 + 3x_2 \geq 2$,

$x_1 + x_2 \geq 1$,

$x_1, x_2 \geq 0$.

c) Solve the following L.P.P graphically:

Maximize $Z = 2x + 5y$

subject to $0 \leq x \leq 4$,

$0 \leq y \leq 3$,

$x + y \leq 6$.

d) Show that all three of the basic solutions of the system $x_1+2x_2+3x_3=6$, $2x_1+x_2+4x_3=4$ exist and they are $\left(0, \frac{12}{5}, \frac{2}{5}\right)$, $(-6, 0, 4)$ and $\left(\frac{2}{3}, \frac{8}{3}, 0\right)$.

e) Find the optimal assignments to find the minimum cost for the cost matrix:

	I	II	III	IV	V
A	6	5	8	11	16
B	1	13	16	1	10
C	16	11	8	8	8
D	9	14	12	10	16
E	10	13	11	8	16

f) Solve graphically or otherwise the game whose pay-off matrix is:

		B			
		B_1	B_2	B_3	B_4
	A_1	8	15	−4	−2
A	A_2	19	15	17	16
	A_3	0	20	15	5

4. Answer any **one** question: 10×1=10

a) i) Solve the following transportation problem:

	D_1	D_2	D_3	D_4	a_i
O_1	10	7	3	6	3
O_2	1	6	8	3	5
O_3	7	4	5	3	7
b_j	3	2	6	4	

ii) Use Charnes Big M-method to solve the L.P.P.:

Maximize $Z = x_1 + 5x_2$

subject to $3x_1 + 4x_2 \le 6,$

$x_1 + 3x_2 \ge 3,$

$x_1, x_2 \ge 0.$ 6+4=10

b) i) Use two-phase method to solve the L.P.P.:

Maximize $Z = 2x_1 + x_2 + x_3$

subject to $4x_1 + 6x_2 + 3x_3 \le 8,$

$3x_1 - 6x_2 - 4x_3 \le 1,$

$2x_1 + 3x_2 - 5x_3 \ge 4,$

$x_1, x_2, x_3 \ge 0.$

ii) Reduce the feasible solution $x_1=2$, $x_2=1$, $x_3=1$ of the system of equations

$$x_1+4x_2-x_3=5$$

$$2x_1+3x_2+x_3=8$$

to a basic feasible solution. 6+4=10

GROUP–B

(Probability Theory)

[Marks : 30]

5. Answer any **four** questions: 1×4=4

a) State classical definition of probability.

b) Prove that $P(A^c)=1-P(A)$.

c) Define mean of a distribution.

d) State Bayes theorem.

e) When two events are said to be stochastically independent?

f) Define random variable.

6. Answer any **four** questions: 2×4=8

a) Find the mean and variance of a random variable that follows Poisson distribution.

b) Let X be a bionomially distributed random variable with its parameters n and p. Assuming n fixed, find the valne of p for which var(X) is maximum.

c) Verify that the following is a distribution function

$$F(x)=\begin{cases}0, \ x<-a \\ \frac{1}{2}\left(\frac{x}{a}+1\right), & -a\le x\le a. \\ 1, \ x>a.\end{cases}$$

d) Find the mathematical expectation of the sum of points on throwing a dice k-times.

e) If the events A and B are independent, then prove that A^c and B^c are also independent.

f) Find the probability of getting at least one 'head' in two throws of a unbiased coin.

7. Answer any **three** questions: 6×3=18

a) State and prove Baye's theorem.

b) An user contains n tickets, numbered 1, 2, ...,n tickets are drawn successively one by one without replacement. If the r-th ticket appears at r-th drawing, then we get a match. Find the probability of at least one match.

c) If X~N(0, 1), then find the density function of e^X.

d) State and prove the approximation of binomial distribution by Poisson distribution.

e) Determine the value of k, such that the function f(x) defined by

$$f(x)=\begin{cases}kx(1-x), & 0<x<1 \\ 0, & \text{elsewhere}\end{cases}$$

is a probability density function. Then find the value of $P\left(X>\frac{1}{2}\right)$. 3+3=6

GROUP–C

(Statistics)

[Marks : 20]

8. Answer any **four** questions: 1×4=4

a) Distinguish between the positive and negative skewness.

b) What is the main difference between absolute and relative measures of dispersion?

c) What do you mean by 'root mean–square deviation'?

d) Give an example of a distribution which have same mean, medium and mode.

e) What is correlation co-efficient?

f) What do you mean by ‘regression line’?

9. Answer any **three** questions: 2×3=6

a) In a partially destroyed laboratory record of an analysis of correlation data, the following results only are legible: Variance of X=9, Regression lines:

$$8X-10Y+66=0,\ 40X-18Y=214.$$

What was the correlation co-efficient between X and Y?

b) If one of the regression co-efficient is greater than unity, prove that the other must be less than unity.

c) Set $X_1, X_2, ..., X_n$ are random sample of size n. Prove that the means of the sample mean is equal to the population mean.

d) Prove that the sum of the squares of the deviations of a set of values is minimum when taken about mean.

e) The first three moments of a distribution about 2 are 1, 16 and 40 respectively. Examine the skewness of the distribution.

10. Answer any **two** questions: 5×2=10

a) Calculate the standard deviation for the following table giving age-distribution of 542 members:

Age in years:	20-30	30-40	40-50	50-60	60-70	70-80	80-90
No. of members:	3	61	132	153	140	51	2

b) For the two frequency distributions given below, the mean calculated from the first is 25.4 and from the second is 32.5.

Class	Distribution–I Frequency	Distribution–II Frequency
10-20	20	4
20-30	15	8
30-40	10	4
40-50	x	2x
50-60	y	y

Find the values of x and y.

c) Calculate the co-efficient of correlation from the following data:

X:	1	2	3	4	5	6	7	8	9
Y:	9	8	10	12	11	13	14	16	15